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Enhanced heat conduction in oscillating viscous flows within parallel-plate channels

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The hydrodynamics of enhanced longitudinal heat transfer through a sinusoidally oscillating viscous fluid in an array of parallel-plate channels with conducting sidewalls is examined analytically. Results show that for fixed frequency the corresponding effective thermal diffusivity reaches a maximum when the product of the Prandtl number and the square of the Womersley number is approximately equal to $\alpha^2 Pr = \pi$. Under such tuned conditions the axial heat transfer achievable is considerable and can exceed that possible with heat pipes by several orders of magnitude. The heat flux between different temperature reservoirs connecting the parallel-plate-channel configuration is shown, under tuned conditions, to be proportional to the first power of both the axial temperature gradient and the flow oscillation frequency and to the square of the tidal displacements. A large value for the fluid density and specific heat is also found to be beneficial when large heat-transfer rates are desired. The process discussed involves no net convection and hence achieves large heat-transfer rates (in excess of 10^6 W/cm^2) without a corresponding net convective mass transfer. A discussion of the physical origin for this new heat-transfer process is given and suggestions for applications are presented.

1. Introduction

It is known that the axial dispersion of contaminants within laminar flows through capillary tubes under both steady (Taylor 1953; Aris 1956) and oscillatory flow conditions (Chatwin 1975; Jaeger 1983; Watson 1983; Joshi 1983) is considerably larger than in the absence of flow. This enhanced transport is produced by the interaction of the radially dependent axial-velocity profile and the corresponding radially varying concentration profile and can lead to effective axial dispersion coefficients orders of magnitude larger than the corresponding value of the molecular-diffusion coefficient. The present author (Kurzweg 1983) has suggested that a similar dispersion process should occur in the heat-transfer area due to the similarity between the diffusion and heat-conduction equations. Indeed, except for some more complicated boundary conditions arising in the thermal problem, the results of the contaminant dispersion problem should be directly applicable. We have most recently confirmed the existence of such an enhanced heat-transfer process in high-frequency oscillatory flow within a capillary bundle connecting two reservoirs maintained at different temperatures (Kurzweg & Zhao 1984). Using water as the working fluid, effective thermal diffusivities some four orders of magnitude larger than the molecular diffusivity value for water, were measured. The corresponding heat-transfer rates are comparable to those achievable with heat pipes and thus suggest that the phenomenon may find important applications in areas requiring the rapid removal of heat without an accompanying net convective mass transfer.

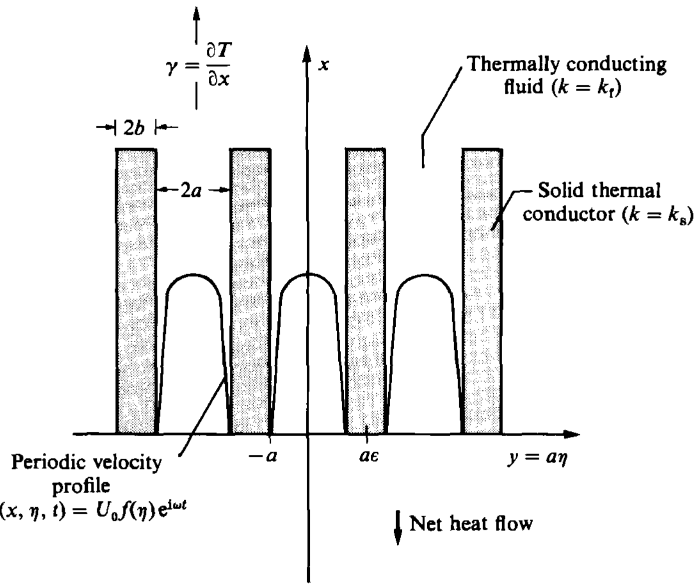

FIGURE 1. Geometry of the parallel-channel flow configuration under investigation.

Our purpose here is to develop a comprehensive theory for this new heat-transfer process. We will confine our attention to strictly oscillatory flows in parallel-plate channels (see figure 1) but, in contrast to our earlier studies, will place no restrictions on the oscillation frequency or the wall conductivity. Of particular interest to us will be a determination of the enhanced axial thermal diffusivity as a function of the Womersley number, the fluid Prandtl number, and the ratio of fluid-to-wall conductivity. As will be shown, the heat-transfer process, at fixed frequency, is most efficient near $\alpha^2 Pr = \pi$, where α is the Womersley number and Pr the Prandtl number. Asymptotic values for the enhanced thermal diffusivity will also be presented.

2. Formulation of the problem

We consider a pressure-gradient-induced periodic viscous flow within a long parallel plate channel array as indicated schematically in figure 1. The ends of the channels terminate in large reservoirs which are maintained at constant but different temperatures so that a time-averaged constant axial temperature gradient $\gamma = \partial T/\partial x$ is maintained both within the thermally conducting fluid and the thermally conducting walls. This problem is analogous to the contaminant-diffusion problem under oscillatory conditions (Chatwin 1975) except that here it will be heat which is transported in a direction opposite to the temperature gradient and, unlike in the contaminant-diffusion case, the time-dependent radial temperature variation will propagate a finite distance into the conducting bounding walls of the channels. The widths of the fluid layers and the solid walls in this 'thermal pump' configuration are taken as $2a$ and $2b$, respectively.

Neglecting end effects, present at points where the channels enter the fluid reservoirs, and assuming laminar-flow conditions, the axial-velocity profile existing in the central channel is represented by the real part of

$$U(\eta, t) = U_0 f(\eta)\, e^{i\omega t} = U_0 \frac{i\lambda}{\alpha^2}\left[1 - \frac{\cosh \sqrt{i}\,\alpha\eta}{\cosh \sqrt{i}\,\alpha}\right] e^{i\omega t}, \tag{1}$$

where $\eta = y/a$ is the non-dimensional coordinate normal to the flow direction, U_0 a representative axial velocity, t the time, $\alpha = a\sqrt{(\omega/\nu)}$ the Womersley number, ν the fluid kinematic viscosity, ρ the fluid density, and $\lambda = |\partial p/\partial x|a^2/\rho U_0 \nu$ the non-dimensional magnitude of the imposed sinusoidal pressure gradient varying in time with an angular velocity ω. The velocity profiles in the other parallel channels are identical in shape and are represented analytically by simply replacing η by $\eta \pm 2n(1+\epsilon)$. Here n is an integer and $\epsilon = b/a$. At large α the velocity profile (1) has an essentially slug-flow character away from the walls and boundary layers of thickness $\delta = \sqrt{2}\, a/\alpha$ existing at the channel walls. For small α the flow has the familiar Poiseuille profile modified by the $\exp \mathrm{i}\omega t$ term.

When dealing with such time-dependent flows it is convenient to introduce the concept of the tidal displacement Δx. This quantity can be defined as the cross-stream-averaged maximum axial distance which the fluid elements travel during one half period of the oscillation. Mathematically the tidal displacement equals

$$\Delta x = U_0 \left| \int_{-\pi/2\omega}^{\pi/2\omega} \mathrm{e}^{\mathrm{i}\omega t}\, \mathrm{d}t \int_0^1 f(\eta)\, \mathrm{d}\eta \right|, \tag{2}$$

which upon integration, using the form for $f(\eta)$ defined in (1), becomes

$$\Delta x = \frac{2U_0 \lambda}{\omega \alpha^2} \left| 1 - \frac{\tanh \sqrt{\mathrm{i}}\, \alpha}{\sqrt{\mathrm{i}}\, \alpha} \right|. \tag{3}$$

Note that for very high frequencies, and hence large α, the term within the absolute-value sign in (3) goes to unity so that Δx becomes $2\,|\partial p/\partial x|/\rho\omega^2$ in this limit. It should be pointed out that the value of Δx is always taken as less than the distance between fluid reservoirs in order to insure that no direct convective mass transfer can occur.

The temperature $T(x, y, t)$ within this channel flow can be determined from a solution of

$$T_t + U_0 f(\eta)\, \mathrm{e}^{\mathrm{i}\omega t}\, T_x = \kappa_\mathrm{f}(T_{xx} + T_{yy}), \tag{4}$$

in the range $0 < \eta < 1$ and

$$T_t = \kappa_\mathrm{s}(T_{xx} + T_{yy}), \tag{5}$$

over the range $1 < \eta < \epsilon = b/a$. Here κ_f and κ_s are the thermal diffusivities of the fluid and the solid conductor, respectively. The viscous-heating term has been neglected in (4) since it is very small for most experimental conditions except those where one deals with high-Prandtl-number fluids such as oils. Typical temperature differences produced by viscous heating are $\Delta T = Pr\,(\omega\,\Delta x)^2/c$, where c is the fluid specific heat. Note that the symmetry of the problem dictates that there can be no heat flow in the y (or η) direction across planes located in the middle of either the fluid channels or the middle of the solid walls. The boundary conditions at the fluid–wall interface at $\eta = 1$ are that both the temperature variation and the heat flux be continuous there. To solve (4) and (5) exactly for the velocity profile under consideration is a formidable task. Fortunately, in the present problem such a general solution will not be necessary when it is realized that in the geometry considered the time-averaged axial temperature gradient has the constant value γ. This suggests that one try a locally valid solution of the form

$$T(x, \eta, t) = \gamma[x + ag(\eta)\, \mathrm{e}^{\mathrm{i}\omega t}], \tag{6}$$

as first proposed by Chatwin (1975). Note that this form for $T(x, \eta, t)$ has a physically realistic locally time-averaged constant axial temperature gradient and also exhibits a time-dependent cross-stream variation in temperature. Kurzweg (1983) has shown that it is not always necessary to use (6) in deriving an effective thermal dispersion coefficient. Substituting this expression into (4) and (5) yields

$$g_{\mathrm{f}}'' - \mathrm{i}\alpha^2 Pr\, g_{\mathrm{f}} = Pe f, \tag{7}$$

and

$$g_{\mathrm{s}}'' - \mathrm{i}\alpha^2 \sigma Pr\, g_{\mathrm{s}} = 0, \tag{8}$$

where the prime denotes differentiation with respect to η, $Pr = \nu/\kappa_{\mathrm{f}}$ is the fluid Prandtl number, $Pe = U_0 a/\kappa_{\mathrm{f}}$ is the Péclet number, $\sigma = \kappa_{\mathrm{f}}/\kappa_{\mathrm{s}}$ is the ratio of fluid to wall thermal diffusivity with the functions g_{f} and g_{s} representing the η-dependent temperature functions as defined by (6) within the fluid and within the channel wall, respectively. The boundary conditions for these functions are $g_{\mathrm{f}}'(0) = 0$ and $g_{\mathrm{s}}'(\epsilon) = 0$ together with the two interfacial conditions $g_{\mathrm{f}}(1) = g_{\mathrm{s}}(1)$ and $\mu g_{\mathrm{f}}'(1) = g_{\mathrm{s}}'(1)$, where $\mu = k_{\mathrm{f}}/k_{\mathrm{s}}$ is the ratio of fluid to wall thermal conductivity. Closed-form solutions for g satisfying these conditions are readily found to be

$$g_{\mathrm{f}}(\eta) = C_1 \cosh \surd(\mathrm{i}\, Pr)\, \alpha\eta + \frac{\lambda Pe}{\alpha^4 Pr\,(Pr-1)} + \frac{\mathrm{i}\, Pe}{\alpha^2(Pr-1)} f(\eta) \tag{9}$$

and

$$g_{\mathrm{s}}(\eta) = C_2 \cosh \surd(\mathrm{i}\sigma\, Pr)\, \alpha(\epsilon-\eta). \tag{10}$$

Here the constants C_1 and C_2 have the values

$$C_1 = \frac{-\lambda Pe}{\alpha^4(Pr-1)\, Pr \cosh \surd(\mathrm{i}\, Pr)\,\alpha} \times \left[\frac{\mu \surd Pr \tanh \surd \mathrm{i}\,\alpha + \surd\sigma \tanh \surd(\mathrm{i}\sigma\, Pr)\,\alpha(\epsilon-1)}{\mu \tanh \surd(\mathrm{i}\, Pr)\,\alpha + \surd\sigma \tanh \surd(\mathrm{i}\sigma\, Pr)\,\alpha(\epsilon-1)}\right], \tag{11}$$

and

$$C_2 = \frac{C_1 \cosh \surd(\mathrm{i}\, Pr)\,\alpha + \dfrac{\lambda Pe}{\alpha^4 Pr\,(Pr-1)}}{\cosh \surd(\mathrm{i}\sigma\, Pr)\,\alpha(\epsilon-1)}. \tag{12}$$

The apparent singularity in these solutions as the Prandtl number approaches unity can easily be resolved by an appropriate limiting procedure.

3. Enhanced thermal diffusivity

Having determined the velocity and approximate temperature distribution within the channels, we are now in the position to calculate the enhanced heat transfer which can be expected to occur from the hot to the cold end of the thermal pump under consideration. Neglecting the small contribution owing to axial thermal conduction in the heat-transfer process, it is clear that an effective averaged thermal diffusivity κ_{e} can be defined by the equality

$$-\kappa_{\mathrm{e}}\gamma = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \mathrm{d}t \int_0^1 [T(x,\eta,t)]_{\mathrm{R}}\, [U_0 f(\eta)\, \mathrm{e}^{\mathrm{i}\omega t}]_{\mathrm{R}}\, \mathrm{d}\eta, \tag{13}$$

where the subscript R refers to the real part of the terms shown. The left-hand side of this expression represents the effective axial thermal flux per unit cross-sectional area and the right side the time-averaged convective thermal flux produced by the interaction of the cross-stream-varying velocity and temperature profiles. In general the real parts of the quantities in this integrand do not average out to zero over time so that there will be a net heat flow although, clearly, the time average of the velocity will be zero so that there can be no net accompanying mass transport. This is an important property of the heat-transfer process studied and suggests many applications including the cooling of radioactive fluids.

Substituting the explicit form for g_f and f into (13) and performing the time integration yields

$$\frac{\kappa_e}{\kappa_f} = -\tfrac{1}{4} Pe \int_0^1 [f\bar{g}_f + g_f \bar{f}]\, d\eta, \tag{14}$$

as the ratio of the enhanced thermal diffusivity to the molecular thermal diffusivity. Here the bars designate the complex conjugate of the functions shown. This form is identical with the result obtained by Watson (1983) provided the function g_f is used to designate the cross-stream variation of contaminant in the related concentration dispersion problem and if the non-penetration condition $g_f'(1) = 0$ is imposed.

An evaluation of the integral given in (14) is most efficiently accomplished via an integration-by-parts approach after introduction of the functions

$$F(\eta) = f(\eta) + \frac{i\lambda}{\alpha^2 Pr}, \tag{15}$$

and

$$G(\eta) = g_f(\eta) + \frac{\lambda Pe}{\alpha^4 Pr}, \tag{16}$$

which can be shown to satisfy the identity

$$f\bar{g}_f + \bar{f} g_f = \frac{1}{i\alpha^2(1+Pr)} [\bar{G}F''' - G\bar{F}''' + \bar{F}G'' - F\bar{G}''], \tag{17}$$

which in turn follows from (7) and (1). Substituting (17) into (14) and integrating by parts yields

$$\frac{\kappa_e}{\kappa_f} = \frac{Pe}{4i\alpha^2(1+Pr)} \{G(1)\bar{F}'(1) - \bar{G}(1)F'(1) + F(1)\bar{G}'(1) - \bar{F}(1)G'(1)\}. \tag{18}$$

It is now a relatively simple matter, using (1), (9), (15) and (16) and their derivatives, together with the definition of the tidal displacement Δx given by (3), to obtain the final expression for the effective (or enhanced) thermal diffusivity of

$$\kappa_e = \omega\,\Delta x^2\, \frac{Pr(1-H)\bar{h} + (1-\bar{H})h + (\bar{h} - \bar{j}\bar{H}) + (h - jH)}{16\alpha^2(Pr^2-1)\left|1 - \dfrac{\tanh\sqrt{i}\,\alpha}{\sqrt{i}\,\alpha}\right|^2}, \tag{19}$$

where

$$h = \sqrt{i}\,\alpha \tanh\sqrt{i}\,\alpha, \tag{20}$$

$$j = \sqrt{(i\,Pr)}\,\alpha \tanh\sqrt{(i\,Pr)}\,\alpha, \tag{21}$$

and

$$H(Pr,\mu,\sigma,\alpha,\epsilon) = \frac{1}{Pr}\left[\frac{\mu\sqrt{Pr}\tanh\sqrt{i}\,\alpha + \sqrt{\sigma}\tanh\sqrt{(i\sigma Pr)}\,\alpha(\epsilon-1)}{\mu\tanh\sqrt{(i\,Pr)}\,\alpha + \sqrt{\sigma}\tanh\sqrt{(i\sigma Pr)}\,\alpha(\epsilon-1)}\right]. \tag{22}$$

4. Evaluation of equation (19)

In evaluating (19) it is convenient to find the non-dimensional ratio $\kappa_e/\omega\,\Delta x^2$ as a function of α for fixed values of Pr, μ, σ, and ϵ. Since both terms contain the angular frequency ω it is clear that the resultant plots must be interpreted as corresponding to a constant-frequency flow. We begin such an evaluation by examining the value of κ_e at very high frequencies where α and $\alpha^2 Pr$ are large quantities. In this limit $h = \sqrt{(\mathrm{i}\alpha)}$ and $j = \sqrt{(\mathrm{i}\,Pr)}\,\alpha$, so that

$$\frac{\kappa_e}{\omega\,\Delta x^2} = \frac{1}{8\sqrt{2}\,\alpha}\left[\frac{Pr\,(Pr+1)\,(\mu+\sqrt{\sigma})-\sqrt{Pr}\,(1+\sqrt{Pr})\,(\mu\sqrt{Pr}+\sqrt{\sigma})}{Pr\,(Pr^2-1)\,(\mu+\sqrt{\sigma})}\right]. \tag{23}$$

This indicates that $\kappa_e/\omega\,\Delta x^2$ is proportional to the inverse first power of the Womersley number. Furthermore, the right-hand side of this expression increases with decreasing Prandtl number for small μ and σ. There is no dependence on wall thickness ($2a\epsilon$) in this high-frequency limit. The reason for this is that the wall penetration distance is of order $\sqrt{(2\kappa_s/\omega)}$ and hence much smaller than the wall thickness. The result (23) is in agreement within a factor of $\frac{1}{2}$ with results from a recently related study (Kurzweg & Zhao 1984) where attention was confined to enhanced heat conduction in tubes for high-frequency pulsating flows. The factor of $\frac{1}{2}$ arises from the different form of (13) when κ_e is expressed in Cartesian or cylindrical coordinates. Since in this limit κ_e is proportional to α, it is clear that at very high frequencies the enhanced thermal diffusivity goes as the square root of frequency and hence will be a monotonic increasing function of ω. Such an asymptotic behaviour has also been found by Watson (1983) in his related study of contaminant dispersion.

The other limit for which (19) lends itself to a rapid evaluation corresponds to low-oscillation frequency where α, $\alpha^2 Pr$, and $(\epsilon-1)\,\alpha\sqrt{(\sigma\,Pr)}$ assume small values. To obtain a non-indeterminate answer in this limit it is necessary to use the four-term approximation

$$\sqrt{\mathrm{i}}\,\alpha\,\tanh\sqrt{\mathrm{i}}\,\alpha = \mathrm{i}\alpha^2[1-\tfrac{1}{3}\mathrm{i}\alpha^2-\tfrac{2}{15}\alpha^4+\tfrac{17}{315}\mathrm{i}\alpha^6]. \tag{24}$$

Some manipulations then lead to the result

$$\frac{\kappa_e}{\omega\,\Delta x^2} = \frac{\alpha^2\,Pr}{8}\left\{\frac{17}{35}-\frac{4\mu}{5[\mu+\sigma(\epsilon-1)]}+\frac{\mu[\mu+\sigma^2(\epsilon-1)^3]}{3[\mu+\sigma(\epsilon-1)]^2}\right\}. \tag{25}$$

In this expression $\kappa_e/\omega\,\Delta x^2$ is proportional to α^2. That is, at small frequencies, the thermal diffusivity κ_e is directly proportional to the square of the oscillation frequency. It is also noted that the right-hand side of (25) increases with increasing μ, σ and ϵ. Both (23) and (25), as well as (19), show that κ_e is proportional to the square of the tidal displacement. A final observation concerning the above limiting forms for $\kappa_e/\omega\,\Delta x^2$ is that they imply the existence of a maximum in $\kappa_e/\omega\,\Delta x^2$ at a finite value of the Womersley number α, since for small α this ratio is proportional to α^2 while at high α it is inversely proportional to α. The observation implies that we are dealing with a tuning process where for fixed frequency ω there is an optimum channel width $2a$ which will maximize $\kappa_e/\omega\,\Delta x^2$ and hence allow a maximum heat to be transported axially.

To determine the exact shape of such 'tuning curves' for fixed Pr, μ, σ and ϵ it is necessary to go back directly to (19) and evaluate this expression as a function of α. We have done this for the special case where the fluid and channel walls have identical thermal properties ($\mu = \sigma = 1$) and the fluid and wall thickness are equal ($\epsilon = 2$). Such a calculation, using a hand calculator, yields the results shown in figure 2 for the three fluid Prandtl numbers of 0.01, 10, and 1000. These values of Pr can be considered as representative values for liquid metals, water and viscous oils, respectively. Note that each of the three curves show the anticipated maximum in $\kappa_e/\omega\,\Delta x^2$ at finite α and that these maxima lie close to the value $\alpha^2 Pr = \pi$. Physically these peaks correspond to the point where the thermal diffusion time of heat from the axis ($\eta = 0$) to the channel wall ($\eta = 1$) of a^2/κ_f just equals one half period π/ω

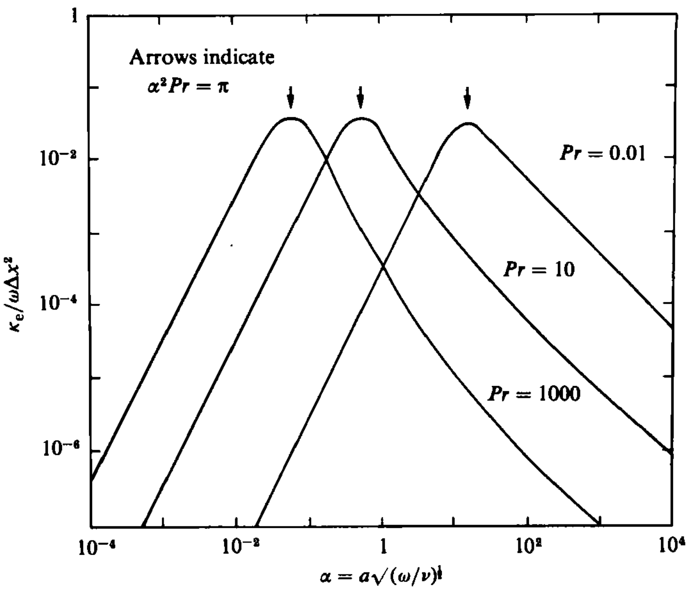

FIGURE 2. Ratio $\kappa_e/\omega\,\Delta x^2$ as a function of Womersley number α for three different Prandtl numbers. The fluid width is equal to the wall thickness ($\epsilon = 2$) and the thermal conductivity and diffusivity of the fluid and wall are equal ($\mu = \sigma = 1$).

of the flow oscillation. That is, there is sufficient time for heat to flow from the fluid core to the walls or vice versa before the temperature reverses itself within the core. At the same time, the time-dependent temperature gradient in the η direction will remain large so as to allow a large heat flow in that direction. Another interesting character of these tuning curves is that their maximum value changes only slightly with change in Prandtl number and that, near these maxima, κ_e is directly proportional to $\Delta x^2\,\omega$ and nearly independent of a and ν. For the values of μ, σ and ϵ used here the ratio of $\kappa_e/\Delta x^2\,\omega$ at its maximum is approximately 0.03. A final interesting feature of (19), as pointed out to me by my colleague, M. Jaeger, is that it also predicts an enhanced thermal diffusivity if the fluid is replaced by solid conducting plates which are oscillated sinusoidally. This limit is found by letting α approach infinity (i.e. zero-thickness boundary layers) but keeping $\alpha^2\,Pr$ finite.

5. Heat conduction between fluid reservoirs

Having determined the general characteristics of the effective thermal diffusivity as a function of Womersley number, we are now in a position to calculate the expected heat transfer between the fluid reservoirs located at the ends of the parallel-channel array. Since we are neglecting the small contribution to the heat-transfer process produced by direct thermal conduction in the x-direction in either the fluid or solid walls, the net heat flux per area for unit-depth channels from the hot to the cold reservoir will be

$$\dot{q} = \frac{\kappa_e \rho c \gamma}{\epsilon} \quad (\mathrm{cal\ s^{-1}\ cm^{-2}}), \tag{26}$$

where ρ and c are the fluid density and specific heat, respectively. The value of κ_e

in this expression is that given by (19). Note that for optimum tuning this result implies that

$$\dot{q} = \text{const} \times \frac{J\rho c\gamma\omega\,\Delta x^2}{\epsilon} \quad (\mathrm{W/cm^2}), \tag{27}$$

where the constant has values depending on μ, σ and ϵ and $J = 4.18$ J/cal is the mechanical equivalent of heat. Interesting features of this result are that the heat transfer is largest for fluids with large ρc, that it is directly proportional to the temperature gradient γ, and that it goes as the first power of the oscillation frequency ω and as the square of the tidal displacement Δx. The values for this enhanced heat flow can be quite large. For example, taking pressurized water with $\rho c = 1$ (cgs units) for a temperature gradient of $\gamma = 10$ °C/cm at an oscillation frequency of 300 rad/s and a tidal displacement of $\Delta x = 100$ cm in a parallel channel array with $\mu = \sigma = 1$ and $\epsilon = 2$, for which the above constant is approximately 0.03, predicts an axial heat transport of 1.8×10^6 W/cm^2. This value is some two orders of magnitude larger than the best values obtainable with liquid-metal heat pipes (Dunn & Reay 1978). Since it is necessary to have large values of $\gamma\rho c$ in order to obtain large heat-flow rates, it may be of advantage in practical heat-transfer devices, based on the present process, to use liquid metals as the heat-transfer fluid as these allow a much larger value of γ although they have smaller values of ρc than water by a factor of about two. The use of gas as a working fluid would lead to much lower heat-transfer rates at the same $\omega\,\Delta x^2$ because ρc is then small.

6. Physical interpretation of the results

The results given by (19) and (26) can be understood in a qualitative sense from pure physical considerations. What occurs in this enhanced thermal conduction process is that large oscillating temperature gradients in the direction normal to the channel walls are produced when the fluid oscillates and an axial temperature gradient is present in the system. During the forward part of the oscillation, hotter fluid within the core causes a heat flow to the colder portions of the fluid within the boundary layers and to the colder solid walls bounding the fluid. The rate of heat flow will be increased the narrower the width of the boundary layer becomes and hence the larger the flow frequency becomes. During the reverse phase of the oscillation, hotter fluid from the boundary layers and the walls will diffuse into the fluid core, which is now colder. The overall effect of this procedure is to pump heat from the hotter to the colder portions of the fluid without an accompanying net convective mass transport. There will be some axial diffusion mass transfer but this will be small for high-Schmidt-number fluids such as water or liquid metals. The analytical prediction that the heat transfer is proportional to the square of the tidal displacement follows from the fact that the radial heat flow is proportional to the product of the representative radial temperature gradient $\gamma\,\Delta x/\delta$ and the surface area per unit depth of $2\Delta x$ available for cross-stream heat transport. Note that this process is strictly laminar and will retain its important characteristic of producing heat transfer without an accompanying convective mass transfer only as long as laminar conditions are maintained. Intermittent turbulent conditions, which may occur in these flows at higher values of $\omega\,\Delta x^2/\nu$ (Merkli & Thomann 1975), would destroy this property. Fortunately, most heat-transfer devices, based on the present concept and running at the optimum of the tuning curves near $\alpha^2\,Pr = \pi$, will require very narrow channels in which viscous effects can be expected to be sufficiently large to prevent the appearance of turbulence. The reason that the present heat-transfer process is less efficient at very low frequencies is that the effective radial temperature gradient for a given Δx becomes smaller owing to the widening of the viscous boundary layer to essentially the half width of the channel (i.e. $\delta \approx a$).

7. Concluding remarks

We have examined the hydrodynamics of the heat-transfer process occurring in oscillating channel flow when the ends of the channels are connected to reservoirs maintained at different temperatures. The enhanced axial thermal diffusivity has been determined and the corresponding axial heat flow rate found. The heat-transfer rates achievable are shown to be very large, exceeding those possible with heat-pipe technology. The transfer process is especially suited for those problems where it is desirable to transport large quantities of heat without an accompanying convective mass transfer. The removal of heat from radioactive fluids or from hazardous chemical solutions would appear to be ideally suited for a heat-transfer device based on the heat-transfer process discussed here.

The physical mechanism for the large axial heat flux achievable in the thermal pumping process considered here is an interchange of heat between the core of the flow and the walls and boundary layers. In effect, the process acts very much as an accelerated molecular conduction process in which the tidal displacement Δx replaces the phonon mean free path Λ. Since the macroscopic distance Δx is orders of magnitude larger than Λ, it has the effect of increasing the effective conduction heat transfer in the axial directions by factors of 10^4 and higher. Applications for this heat-transfer process are numerous, ranging from devices in which heat is is removed from the core of a nuclear reactor without an accompanying mass transfer to an accelerated cooling device for removing heat in combustion processes. It should be pointed out that the process works best for fluids with large ρc and hence would not be as effective for transport of heat in gases.

Remaining work for a further understanding of the heat-transfer process discussed here would be a complete integration of the heat-conduction equations (4) and (5) without use of the Chatwin approximation (6) and consideration of the nonlinear friction heating term in (4) not dealt with here. It is expected that such a calculation would yield details of the time-dependent heat-transfer process during each phase of an oscillation cycle, but would not be expected to yield results very different from the present when averaged over time and if Pr and $\Delta x\, \omega$ are kept small. Finally, it is suggested that further experiments be conducted on the process considered here as a supplement to those reported earlier (Kurzweg & Zhao 1984). The role of turbulence in such oscillating flows at high ω and Δx should receive special attention as well as the recognition that inertia forces can become large at higher frequencies of oscillation.

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